

A Letter of Dr. John Wallis to Samuel Pepys Esquire, relating to some supposed Imperfections in an Organ.

Mr. *Harris* an Organ-maker (whom I find, by the little discourse I had with him, to be very well skilled in his profession) was lately with me, as by direction from you, to ask my opinion about perfecting an Organ, in a point wherein he thinks it yet Imperfect.

'Tis an honour you please to put upon me, to think my opinion considerable in a thing wherein I am so little acquainted as that of an Organ.

I do not pretend to be perfectly acquainted with the Structure of an Organ, its several Parts, and the Incidens thereunto; Having never had Occasion and Opportunity to inform my self particularly therein. And, for the same reason, many of the Words, Phrases, Forms of Speech, and Terms of Art, which are familiar to Organists and Organ-makers, are not so to me. Which therefore I shall wave; (For till we perfectly understand one anothers Language, it is not easy to speak intelligibly;) and apply my self directly to what is particularly proposed.

This (I take it) is evident; That each *Pipe* in the Organ is intended to express a distinct *Sound* at such a *Pitch*; That is, in such a determinate Degree of *Gravity* or *Acuteness*; or (as it is now called) *Flatness* or *Sharpness*. And the *Relative* or *Comparative* Consideration of Two (or more) such Sounds or Degrees of Flatness and Sharpness, is the ground of (what we call) *Concord* and *Discord*; that is, a Soft, or Harsh, coincidence.

Now, concerning this, there were amongst the Ancient Greeks, Two (the most considerable) *Sects* of Musicians: the *Aristoxenians*, and the *Pythagorians*.

They both agreed thus far; That *Dia-tessarion* and *Dia-pente*,

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do

do together make-up *Dia-pason* ; that is (as we now speak) a *Fourth* and *Fifth* do together make an *Eighth* or *Octave*: And, the *Difference* of those two (of a *Fourth* and *Fifth*) they agreed to call a *Tone* ; which we now call a *Whole note*.

Such is that, (in our present Musick,) of *La Mi*, (or as it was wont to be called, *Re Mi*.) For *La fa sol la*, or *Mi fa sol la*, is a perfect *Fourth*: And *La fa sol la mi*, or *La mi fa sol la*, is a perfect *Fifth*: The *Difference* of which, is *La mi*. Which is, what the Greeks call, the *Diazeuctick Tone* ; which doth *Dis-join* two *Fourths* (on each side of it ;) and, being added to either of them, doth make a *Fifth*. Which was, in their Musick, that from *Mese* to *Paramese* ; that is in our Musick, from A to B: supposing *Mi* to stand in *B fa b mi*, which is accounted its *Natural* position.

Now, in order to this, *Aristoxenus* and his Followers, did take, that of a *Fourth*, as a *Known Interval*, by the judgement of the Ear ; and, that of a *Fifth*, likewise ; And consequently, that of an *Octave*, as the *Aggregate* of both ; and that of a *Tone*, as the *Difference* of those Two.

And this of a *Tone* (as a *known Interval*) they took as a *common Measure*, by which they did estimate other Intervals. And accordingly they accounted a *Fourth* to contain *Two Tones and an half* ; a *Fifth* to contain *Three Tones and an half* ; and consequently an *Eighth* to contain *Six Tones*, or *Five Tones and two Half-tones*. And it is very near the matter, though not exactly so.

And at this rate we commonly speak at this day ; supposing an *Octave* to consist of *Twelve Hemitones*, or Half-notes. (Meaning thereby, somewhat near so many half-notes :) But, when we would speak more Nicely, we do not take those supposed *Half-notes* to be exactly *Equal*, or each of them just the *Half* of a *Full-note*, such as is that of *La-mi*.

Pythagoras and those who follow him, not taking the Ear alone to be a competent Judge in a case so nice ; chose to distinguish these, not by *Intervals*, but by *Proportions*. And accordingly they accounted that of an *Octave*, to be, when the degree of Gravity or Acuteness of the one Sound to that of the other, is *Double*, or as 2 to 1 ; that of a *Fifth*, when it is *Sesqui-alter*, or as 3 to 2 ; that of a *Fourth* when *Sesqui-tercian*, or as 4 to 3. Accounting That, the Sweetest proportion, which is expressed in the Smallest Numbers ; and therefore (next to the *Unisone*) that
of

of an *Octave*, 2 to 1; then that of a *Fifth*, 3 to 2; and then that of a *Fourth*, 4 to 3.

And thus, that of a *Fourth* and *Fifth*, do together make an *Eighth*; For $\frac{4}{3} \times \frac{3}{2} = \frac{4}{2} = \frac{2}{1} = 2$. That is, *four thirds* of *three halves*, is the same as *four halves*, that is *Two*. Or (in other words to the same sense) the proportion of 4 to 3, compounded with that of 3 to 2, is the same with that of 4 to 2, or 2 to 1. And, consequently, the *Difference* of those *Two*, which is that of a *Tone* or *Full-Note*, is that of 9 to 8. For $(\frac{4}{3})^{\frac{3}{2}} (\frac{3}{2})$; that is, *three halves* divided by *four thirds*, is *nine eights*; or, if out of the proportion of 3 to 2, we take that of 4 to 3; the Result is that of 9 to 8.

Now, according to this Computation, it is manifest, That an *Octave* is somewhat less than *Six Full-notes*. For (as was first demonstrated by *Euclide*, and since by others) the Proportion of 9 to 8, being six times compounded, is somewhat more than that of 2 to 1. For $\frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8} = \frac{531441}{262144}$, is more than $\frac{524288}{262144} = \frac{2}{1}$.

This being the Case; they allowed (indisputably) to that of the *Dia-zeutick Tone* (*La mi*), the full proportion of 9 to 8; as a thing not to be altered; being the Difference of *Dia-pente* and *Dia-tesaron*, or the *Fifth* and *Fourth*.

All the Difficulty, was, How the remaining *Fourth* (*Mi fa sol la*) should be divided into three parts, so as to answer (pretty near) the Aristoxenians *Two Tones and an Half*; and might, altogether, make up the proportion of 4 to 3; which is that of a *Fourth* or *Dia-tesaron*.

Many attempts were made to this purpose: And, according to those, they gave Names to the different *Genera* or *Kinds* of *Musick*, (the *Diatonick*, *Chromatick*, and *Enarmonick* Kinds,) with the several *Species*, or lesser Distinctions under those *Generals*. All which to enumerate, would be too large, and not necessary to our business.

The first was that of *Euclide* (which did most generally obtain for many ages:) Which allows to *Fa sol*, and to *Sol la*, the full proportion of 9 to 8; And therefore to *Fa sol la* (which we call the *greater Third*;) that of 81 to 64. (For $\frac{9}{8} \times \frac{9}{8} = \frac{81}{64}$.) And, consequently, to that of *Mi fa* (which is the Remainder to a *Fourth*) that of 256 to 243. For $(\frac{81}{64})^{\frac{3}{2}} (\frac{256}{243})$; that is, if out of the proportion of 4 to 3, we take that of 81 to 64, the Result is that of 256 to 243. To this they gave the name of *Limma* ($\lambda\epsilon\acute{\iota}\mu\mu\alpha$)

that is, the *Remainder* (to wit, over and above *Two Tones.*)
 But, in common discourse (when we do not pretend to speak
 nicely, nor intend to be so understood) it is usual to call it an
Hemitone or *Half-Note* (as being very near it) and, the other,
 two *Whole-Notes*. And this is what *Ptolemy* calls *Diatonum*
Ditonum, (of the *Diatonick* kind with *Two full Tones.*)

Against this, it is objected (as not the most convenient Divi-
 sion,) that the Numbers of 81 to 64, are too great for that of
 a *Ditone* or *Greater Third*: Which is not *Harsh* to the Ear ;
 but is rather *Sweeter* than that of a single *Tone*, who's propor-
 tion is 9 to 8. And in that of 256 to 243, the Numbers are yet
 much greater. Whereas there are many proportions (as $\frac{5}{4}$, $\frac{6}{5}$, $\frac{7}{6}$, $\frac{8}{7}$,)
 in smaller numbers than that of 9 to 8 ; of which, in this divi-
 sion, there is no notice taken.

To rectify this, there is another Division thought more con-
 venient ; which is *Ptolemy's Diatonum Intensum* (of the *Dia-
 tonick* Kind, more *Intense* or Acute than that other..) Which,
 instead of *Two Full tones* for *Fa sol la* ; assigns (what we now
 call) a *Greater* and a *Lesser* Tone ; (which, by the more nice Mu-
 sicians of this and the last Age, seems to be more embraced ;)
 Assigning to *Fa sol*, that of 9 to 8 (which they call the *Greater*
Tone :) and to *Sol la*, that of 10 to 9, (which they call the *Lesser*
Tone :) And therefore to *Fa la* (the *Ditone* or *Greater Third*)
 that of 5 to 4. (For $\frac{10}{9} \times \frac{9}{8} = \frac{10}{8} = \frac{5}{4}$.) And consequently, to
Mi fa (which is remaining of the *Fourth*) that of 16 to 15. For
 $\frac{5}{4} \frac{4}{3} (\frac{16}{15})$. That is ; if out of that of 4 to 3, we take that of 5 to
 4, there remains that of 16 to 15.

Many other waies there are (with which I shall not trouble
 you at present) of dividing the *Fourth* or *Dia-tessaron*, or the
 proportion of 4 to 3, into three parts, answering to what (in a
 looser way of Expression) we call an *Half-note*, and two *Whole-
 notes*. But this of $\frac{16}{15} \times \frac{9}{8} \times \frac{10}{9} = \frac{4}{3}$, is that which is now received as
 the most proper. To which therefore I shall apply my discourse.
 Where $\frac{16}{15}$ is (what we call) the *Hemitone* or *Half-note*, in *Mi fa* ; $\frac{9}{8}$
 that of the *Greater-Tone*, in *Fa sol* ; and $\frac{10}{9}$ the *Lesser-Tone*, in *Sol la*.

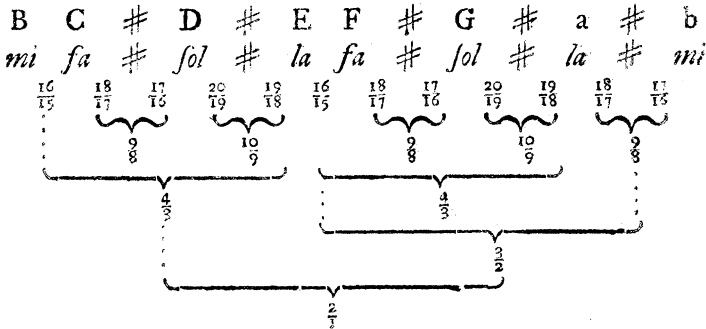
Onely with this addition ; That each of those Tones, is (upon
 occasion) by *Flats* and *Sharps* (as we now speak) divided into
 two *Hemitones* or *Half-notes* : Which answers to what by the
 Greeks was called *Mutatio quoad Modos* (the change of Mood ;)
 and what is now done by removing *Mi* to another Key. Namely

$$\frac{9}{8} = \frac{18}{16} = \frac{18}{17} \times \frac{17}{16} ; \text{ and } \frac{10}{9} = \frac{20}{18} = \frac{20}{19} \times \frac{19}{18} .$$

Thus,

Thus, by the help of *Flats* and *Sharps* (dividing each *Whole-note*, be it the Greater or the Lesser, into two *Half-notes*, or what we call so,) the whole *Octave* is divided into Twelve Parts or Intervals (contained between Thirteen Pipes) which are commonly called *Hemitones* or *Half-notes*. Not, that each is precisely *Half a Note*, but somewhat near it, and so called. And I say, by *Flats* and *Sharps*; For sometime the one, sometime the other, is used. As, for instance, a *Flat* in D, or a *Sharp* in C, do either of them denote a *Midling Sound* (tho' not precisely in the Midst) between C and D; Sharper than C, and Flatter than D

Accordingly; supposing *Mi* to stand in B *fa* b *mi* (which is accounted its Natural feat) the Sounds of each Pipe are to bear these proportions to each other, viz.



And so in each Octave successively following. And if the Pipes in each Octave be fitted to sounds in these proportions of Gravity & Acuteness; it will be supposed (according to this Hypothesis) to be perfectly proportioned.

But, instead of these successive proportions for each Hemitone; it is found necessary (if I do not mistake the practice) so to order the 13 Pipes (containing 12 Intervals which they call Hemitones) as that their Sounds (as to Gravity & Acuteness) be in Continual Proportion, (each to its next following, in one and the same Proportion;) which, all together, shall compleat that of an Octave or Diapason, as 2 to 1. Whereby it comes to pass, that each Pipe doth not express its proper Sound, but very near it, yet somewhat varying from it, Which they call *Bearing*. Which is somewhat of Imperfection in this Noble Instrument, the Top of all.

It may be asked, Why may not the Pipes be so ordered, as to have their Sounds in just Proportion, as well as thus *Bearing* ?

I answer, It might very well be so, if all Musick were Compos'd to the same *Key*, or (as the Greeks call it) the same *Mode*. As, for instance, if, in all Compositions, *Mi* were alwaies placed in *B fa b mi*. For then the Pipes might be ordered in such proportions as I have now designed.

But Musical Compositions are made in great variety of *Modes*, or with great diversity in the *Pitch*. *Mi* is not always placed in *B fa b mi*; but sometimes in *E la mi*; sometimes in *A la mi re*, &c. And (in summe) there is none of these 12 or 13 Pipes but may be made the Seat of *Mi*. And if they were exactly fitted to any one of these cases, they would be quite out of order for all the rest.

As, for instance; If *Mi* be removed from *B fa b mi* (by a *Flat* in *B*) to *E la mi*: Instead of the Proportions but now designed, they must be thus ordered;

B	#	C	#	D	#	E	F	#	G	#	a	b								
<i>fa</i>	#	<i>sol</i>	#	<i>la</i>	#	<i>mi</i>	<i>fa</i>	#	<i>sol</i>	#	<i>la</i>	<i>fa</i>								
$\frac{18}{17}$		$\frac{17}{16}$		$\frac{20}{19}$		$\frac{19}{18}$		$\frac{18}{17}$		$\frac{16}{15}$		$\frac{18}{17}$		$\frac{17}{16}$		$\frac{20}{19}$		$\frac{19}{18}$		$\frac{16}{15}$

Where 'tis manifest, that the removal of *mi* doth quite disorder the whole series of Proportions. And the same would again happen, if *mi* be removed from *E* to *A* (by another *Flat* in *E*) And again if removed from *A* to *D*. And so perpetually.

But the Hemitones being made all Equal; they do indifferently answer all the positions of *Mi* (though not exactly to any :) Yet nearer to some than to others. Whence it is, that the same Tune sounds better at one *Key* than at another.

It is asked, Whether this may not be remedied; by interposing more Pipes; and thereby dividing a Note, not only (as now) into *Half-notes*, but into *Quarter-notes* or *Half-quarter-notes*, &c.

I answer; It may be thus remedied in part; (that is, the Imperfection might thus be somewhat Less, and the Sounds somewhat nearer to the just Proportions :) but it can never be exactly true; so long as their Sounds (be they never so

many) be in continual proportion; that is, each to the next subsequent in the same Proportion.

For it hath been long since Demonstrated, that there is no such thing as a *just Hemitone* practicable in Musick, (and the like for the division of a Tone into any number of *Equal* parts; three, four, or more.) For, supposing the Proportion of a *Tone* or *Full-note*, to be $\frac{9}{8}$ (or, as 9 to 8;) that of the *Half-note* must be as $\sqrt{9}$ to $\sqrt{8}$ (as the *Square-root* of 9 to the *Square-root* of 8; that is, as 3 to $\sqrt{8}$, or 3 to $2\sqrt{2}$;) which are *Incommensurable* quantities. And that of a *Quarter-note*, as $\sqrt{99}$ 9 to $\sqrt{99}$ 8, (as the *Biquadrate* root of 9, to the *Biquadrate* root of 8,) which is yet more *Incommensurate*. And the like for any other number of *Equal* parts. Which will therefore never fall-in with the Proportions of Number to Number.

So that this can never be perfectly adjusted for all Keys (without somewhat of Bearing) by multiplying of Pipes; unless we would for every Key (or every different Seat of *Mi*) have a different Set of Pipes, of which this or that is to be used, according as (in the Composition) *Mi* is supposed to stand in this or that Seat. Which vast number of Pipes (for every Octave) would vastly increase the Charge. And (when all is done) make the whole impracticable.

These are my present thoughts, of the Question proposed to me, and upon these grounds.

You will please to excuse me for the trouble I give you of so long a Letter.

I thought it necessary, to give a little intimation of the Ancient Greek Musick compared with what is now in practise; which is more the same than most men are aware of: though the Language be very different. But I was not to be large in it. Those who desire to know more of it; may see my thoughts more at large, in that Appendix which I have added at the end of my Edition of Ptolemy's Harmonicks in Greek and Latin.

The two Eminent Sects amongst them, the *Aristoxenian* and the *Pythagorian*, differ much at the same rate as doth the Language of our ordinary practical Musicians, and that of those who treat of it in a more Speculative way.

Our Practical Musicians talk of *Notes* and *Half-notes*, just as the *Aristoxenians* did; as if the *Whole Notes* were all Equal; and the *Half-notes* likewise each the just Half of a Whole Note.

And

And thus it is necessary to suppose in the Pipes of an Organ; which have each their determinate Sound and not to be corrected, in their little Inequalities, as the Voice may be by the guidance of the Ear.

But *Pythagoras*, and those who follow him found (by the Ear) that this Equality of Intervals would not exactly answer the Musical Appearances, in Concords and Discords: just as our Organists and Organ-makers be now aware; that their Pipes at equal Intervals do not give the just desired Harmony, without somewhat of *Bearing*, that is, of some little variation from the just Sound.

The *Pythagorians*, to help this, changed the notion of *Equal Intervals* into that of *due Proportions*. And this is followed by *Zarlino*, *Kepler*, *Cartes*, and others who treat of Speculative Musick in this and the last Age. And though they speak of Notes and Half-notes (in a more gross way) much as others do, yet declare themselves to be understood more nicely.

And though our present *Gam-ut* take no notice of this little diversity; yet, in Vocal Musick, the Ear directs the Voice to a more just proportion. And, in String Musick, it may in like manner be helped by straining and slackening the Strings, or moving the Frets. But, in Wind Musick, the Pipes are not capable of such correction; and therefore we must be content with some little irregularity therein; that so they may tolerably answer (though not exactly) the different Compositions according to the different placing of *Mi* in the *Gam-ut*.

Now the Design of Mr. *Harris* seems to be this; either (by multiplying intermediate Pipes) to bring the Organ to a just Perfection: Or else (if that cannot be done) to rest content with the little Imperfection that is; which though, by more Pipes, it may be somewhat abated, yet cannot be perfectly remedied. And in this I think we must acquiesce.

I am

SIR,

Yours to serve you

Oxford June
27. 1698.

JOHN WALLIS,